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Denoising of Complex Signals using Multi band Complex Wavelets with Improved Thresholding

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Abstract — The dual-tree complex wavelet transform (DT-CWT) which utilizes two 2- band discrete wavelet transform (DWT) was recently extended to M- band. In this paper we provide a simple construction method for an M-band DT-CWT, with M = r^d where r, d Z. In particular, we show how to extend a given r- band DT-CWT to an r^d – band one. For convenience, the case where r = 2, d = 2 is considered. However, the scheme can be extended to general {r, d} pairs straightforwardly .There are so many methods to denoise complex noisy signals, but this paper proposes an improved threshold method (soft thresholding with improved thresholding rule) used with M-band DTCWT to Denise the complex signals. Finally, the results obtained using the proposed algorithm is compared with the 2-band DTCWT algorithm.

Index Terms — Wavelets, complex wavelets, 2- band dual- tree, M-band dual tree, Hilbert transform pairs, thresholding, Improved thresholding method.

I. INTRODUCTION

The discrete dual-tree complex wavelet transform (DT-CWT) [1] provide approximate shift-invariance and directionality. The DT-CWT achieves these properties by employing two discrete wavelet associated with second DWT is the Hilbert transform of the first. This scheme was extended to M-band orthonormal wavelet bases in [2], the transform in [2] employs two M-band discrete wavelet transforms where the wavelet associated with the two transforms from Hilbert transform pairs.

It is well known to extend a 2-channel perfect reconstruction (PR) filter bank (FB) into an M-channel PR FB using tree structured FB (with M = 2^d). A tree- structured FB also allows one to extend a 2-band DWT into an M-band DWT; M-band wavelet transforms of that type are often called wavelet packets [3].

However, it is not previously known how to properly extend a 2- band DT-CWT (with M = r^d). In particular, it will be shown how to obtain an FIR 4-band DT-CWT from an FIR 2-band DT-CWT).

This construction can be extended to other {r, d} values straightforwardly.

This paper is organized as follows. In section II, implementation of the 2-Band DTCWT based on complex wavelets is provided. In section III, the necessary and sufficient condition to implement 4-Band DTCWT is discussed. In section IV, the complex signal denoising methods and the proposed improved thresholding methods are discussed. In section V, experimental results are given. At last, section VI, gives some conclusions.

II. THE 2-Band DUAL-TREE CWT

The 2-Band dual tree CWT employs two real DWTs as shown in Figures 1(a) and 1(b); the first DWT gives the real part of the transform while the second DWT gives the imaginary part. Let $h_0(n), h_1(n)$ denote the low pass, high pass filter pair for the upper synthesis FB and let $g_0(n)$, $g_1(n)$ denote the low pass and high pass filter pair for the lower synthesis FB.

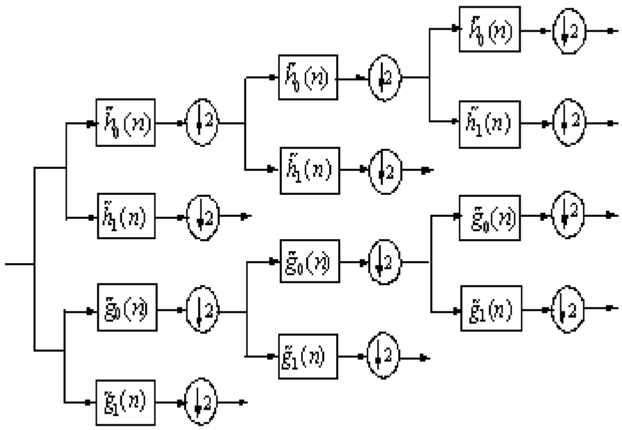

Fig 1(a) Analysis FB for the 2 band DT-CWT

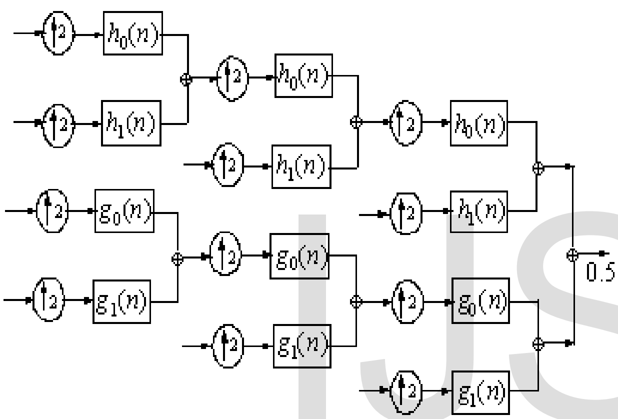

Fig 1(b) Synthesis FB for the 2 band DT-CWT

The inverse of the dual-tree CWT is as simple as the forward transform. To invert the transform, the inverse of each of the two real DWTs are used to obtain two real signals. These two real signals are then averaged to obtain the final output.

To overcome the DWT disadvantages the filters in one tree must provide delays that are *half a sample* different (at each filter's input rate) from those in the opposite tree [4].

$$g_0(n) \approx h_0(n-0.5) \quad \Rightarrow \quad \psi_g(t) = \mathcal{H}\{\psi_h(t)\} \qquad (1)$$

III. THE 4-Band DUAL-TREE CWT

It is well known that 2-band wavelet bases employ approximation spaces v_i which can be decomposed into a higher level approximation space v_{i+1} and a detail space w_{i+1} as

$$v_i = v_{i+1} \oplus w_{i+1} \qquad (2)$$

Where $\oplus$ denotes a direct sum of the vector spaces. The 2- band dual complex wavelet transform asks for a second set of approximation spaces $v_i^{'}$ and the associated orthogonal wavelet spaces $w_i^{'}$, such that the wavelets $\psi(t)$ and $\psi^{'}(t)$ form a Hilbert transform pair.

Similarly, the M-band wavelet transform employs approximation spaces v_i satisfying.

$$v_i = v_{i+1} \oplus w_{i+1}^{'} \oplus .... \oplus w_{i+1}^{M-1} \qquad (3)$$

The M-band DT-CWT is constructed [5,6] by finding a second set of approximation spaces $v_i^{'}$ and wavelet spaces $w_i^{'k}$ such that the associated wavelet functions $\psi^{k}(t)$ and $\psi^{'k}(t)$ from Hilbert transform pairs, for $k \in \{1,2,....\text{M-1}\}$. In the following, we will demonstrate how to construct an r^{d}-band DT- CWT given an r-band DT-CWT. For convenience we will concentrate on the {r = 2, d = 2} case, yielding a 4-band DT-CWT, but the procedure can be easily adapted to general {r, d} pairs.

Suppose we are given a 2-channel orthonormal filter bank $\{h_0^{(2)}(n), h_1^{(2)}(n)\}$ and its associated scaling function $\phi^{(2)}(t)$ and wavelet $\psi^{(2)}(t)$. Suppose we are also given a second 2- cannel filter bank $\{h_0^{'(2)}(n), h_1^{'(2)}(n)\}$ and its associated scaling function $\phi^{'(2)}(t)$ and wavelet $\psi^{'(2)}(t)$, where $\psi^{'(2)}(t)$ is the Hilbert transform of $\psi^{(2)}(t)$, i.e.

$$\psi^{'(2)}(\omega) = j\,\text{sgn}(\omega)\psi^{(2)}(\omega) \qquad (4)$$

Where 'sgn' denotes the signum function. That is, we are given a 2-band 'dual-tree' complex wavelet transform where the complex wavelet $\psi^{(2)}(t) + j\psi^{'(2)}(t)$ is analytical.

Now we would like to construct a 4-band complex wavelet transform. To that end, suppose that $\{f_0(n), f_1(n)\}$ is another 2-channel orthonormal filter bank. We can then obtain a 4-channel orthonormal filter bank. Our aim is to construct a second wavelet packet transform so that the wavelets (associated with the two wavelet transforms) from Hilbert transform pairs.

SUFFICIENCY CONDITIONS FOR THE 4-BAND DT-CWT

Sufficient conditions are given for two *M*-band filter banks so that the associated wavelets form Hilbert transform pairs. The 4-band DT-CWT developed here satisfies these conditions.

$$H^{'}(e^{j\omega}) = e^{-j\theta_k(\omega)} H_k(e^{j\omega})$$

Where

$$\theta_0(\omega) = \begin{cases} 1.5\omega \ \ if\, \omega \in [0, \pi/2) \\ 1.5\omega - \omega \ \ \ for \omega \in [0, \pi/2) \end{cases}$$

and

$$\theta_k(\omega) = 0.5\pi - 0.5\omega \ \ \ for \omega \in [0, \pi), k \in \{1, 2, 3\}$$

These are exactly the sufficiency conditions for Hilbert transform pairs of wavelets for the 4-band case, provided in [6].

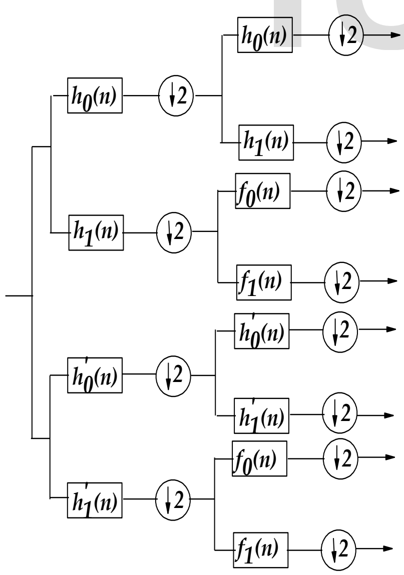

Fig 2 (a) Analysis FB for the 4 band DT-CWT

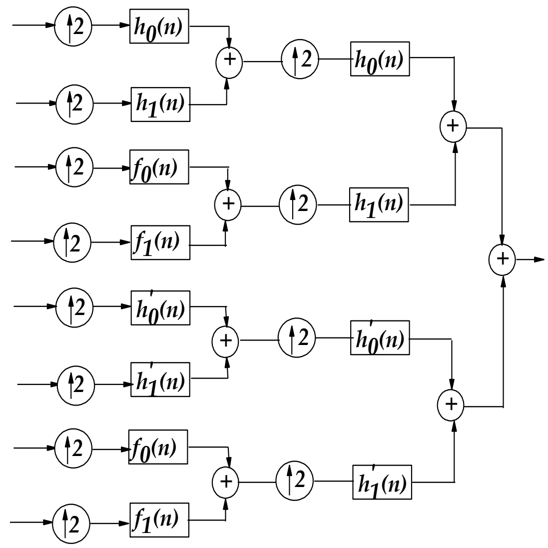

Fig 2 (b) Synthesis FB for the 4 band DT-CWT

Thus, the new wavelets from Hilbert pairs if we set $f_k^{'}(n) = f_k(n)\ for\ k \in \{0,1\}$.Consequently, the dual of the tree is obtained by simply replacing $h_k(n)$ by $h_k^{'}(n)$. This method generates a 4-band dual-tree complex wavelet transform. Here we are using q-shift filters [7] of length 14 are used as $h_k(n)$ and $h_k^{'}(n)$, $f_k(n)$ is set equal to $h_k(n)$ for $k \in \{0,1\}$.

IV. SIGNAL DENOISING

Having a sampled noisy complex signal $s = (s_1, s_2, ..., s_N) \in C^N$ given by

$$s_n = f(x_n) + \sigma z_n, \qquad n = 1, 2, ..., N \tag{5}$$

Where the $z_n = z_{1n} + iz_{2n}$ are identically and independently distributed standard complex Gaussian random variables. Their distribution is $(z_{1n}, z_{2n})' \sim \text{Normal}(0,1)$.

We use the generalized hard thresholding function to complex values as:

$$\hat{w}_j = \begin{cases} w_j, & |w_j| \geq \lambda \\ 0, & |w_j| < \lambda \end{cases} \tag{6}$$

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The generalized soft thresholding function to complex values is:

$$\hat{w}_j = \begin{cases} \operatorname{sgn}(w_j)\left(\left|w_j\right| - \lambda\right), & \left|w_j\right| \geq \lambda \\ 0, & \left|w_j\right| < \lambda \end{cases} \quad (7)$$

Where λ is the threshold, $\lambda = \hat{\sigma}\sqrt{2\log N}$ is the threshold. The robust estimate of the noise level $\hat{\sigma}$ on is given by $\hat{\sigma} = \frac{\text{median}\left(\left|w_n\right| : n = 1, 2, \ldots, N/2\right)}{0.6745}$; here w_n are detail coefficients at the finest level

A. .The Defects of Wavelet Thresholding Methods.

Hard and soft thresholding methods have got better results in de-noising, but they also have some defects:

1) In the hard-thresholding case, the estimated wavelet coefficients $\hat{w}$ are not continuous at position $\pm\lambda$. It may lead to oscillation of the reconstructed signal.
2) In the soft-thresholding case, when $|w| \geq \lambda$, there are constant deviations between $\hat{w}$ and w, which reduces the amplitudes of the reconstructed signal.

B. Improved Wavelet Thresholding Denoising Method

To overcome the defects of hard- and soft-thresholding de-noising methods, an improved thresholding is defined in this paper as follows:

$$D_I^{\lambda}(w) = \begin{cases} \frac{w_j}{\left|w_j\right|}\left(\left|w_j\right| - \beta^{(\lambda_j - w_j)} \cdot \lambda_j\right), & \left|w_j\right| \geq \lambda_j \\ 0, & \left|w_j\right| < \lambda_j \end{cases} \quad (8)$$

Where $\beta \in R^+$ and $\beta > 1$, w_j are the wavelet coefficients at level *j*. Because of the magnitudes of the wavelet coefficients are related to the Gaussian white noise, decrease as the scale *j* increases, we chose

$$\lambda_j = \sigma\sqrt{2\log N} / \log(j+1) \quad (9)$$

Equation (8) will be equivalent to hard-thresholding when $\beta \to \infty$ and will be equivalent to soft-thresholding when $\beta \to 1$. Therefore, the improved thresholding could be regarded as a compromising between the hard- and soft- thresholding.

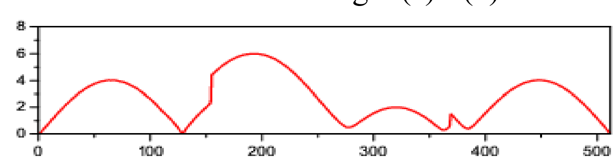
So the improved thresholding de-noising method presented in this paper could choose appropriate β by trial-and-error to satisfy the request of de-noising.

V. EXPERIMENTAL RESULTS

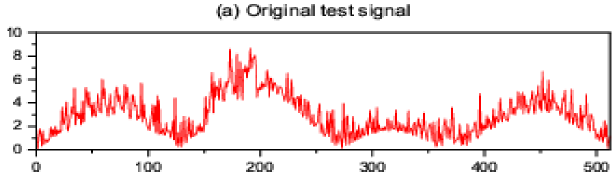
In this paper, at first, the complex test signal is generated by using two real test signals with "Heavy sine" signal as real part and ''Doppler signal'' as imaginary part. This complex test signal is intercepted to be the original test signal. The length of the original test signal (i.e., the number of the sample points) is $N = 512$ (See Fig. 2(a)). Complex Gaussian white noise is added to the original test signal at random. So the noisy test signal with SNR of 10 dB is obtained and shown in Fig. 2(b). The test signal is processed using two different algorithms, respectively (1) Complex data is processed using traditional 2 band DT CWT and (2) Complex data processed using 4 band DT CWT.

For the both DT CWT algorithm Hilbert transform pairs of wavelet bases with 14 coefficients are used (generated with parameters K=4 and L=3). The both DT CWT are performed with hard-thresholding, soft-thresholding and improved thresholding respectively to denoise the noisy test signal. In the two algorithms with improved thresholding, the coefficient β is chosen as β =1.

The denoised test signals by using the universal threshold selection with hard and soft thresholding along with the improved threshold de-noising with three methods are shown in Fig. 2(a)-2(h).



(a) Original test signal



(b) Noisy test signal

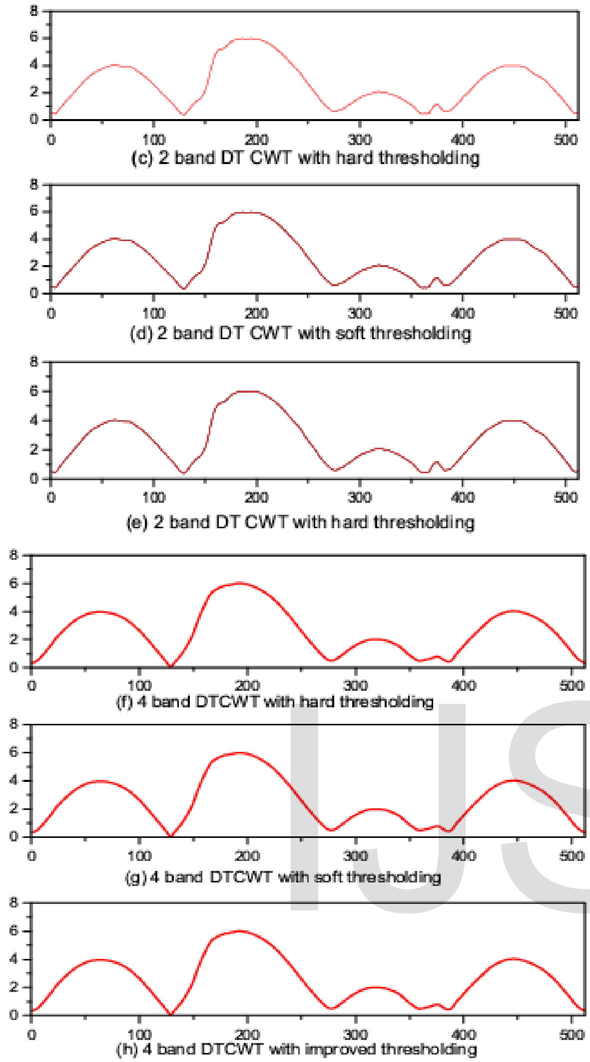


Fig 2. Denoised test signals by using different de-noising methods

From Fig. 2(h), it is clear that the improved thresholding de-noising method using 4 band DT CWT is giving better results.

Table 1 gives the SNR of denoised signals by using different de-noising methods. It can be found that the denoised signal obtained using 4 band DT CWT has higher SNR than the 2 band DTCWT with the improved thresholding has the highest SNR among all methods and is more similar to the original test signal.

TABLE 1

SNR OF DENOISED TEST SIGNALS USING DIFFERENT DENOISING METHODS

Thresholding	Threshold selection rule	2 band DT CWT	4 band DT CWT
Hard	Rigrsure	22.03	23.23
	Heursure	22.21	23.28
	Sqtwolog	22.67	23.56
	Minimaxi	22.34	23.39
Soft	Rigrsure	22.10	23.40
	Heursre	22.23	23.54
	Sqtwolog	22.92	23.66
	Minimaxi	22.98	23.87
Improved	**Improved**	**23.12**	**24.18**

VI. CONCLUSION

We have proposed an extension of existing works on Hilbert transform pairs of dyadic orthonormal wavelets to the M-band case. The 2 band DT CWT with the traditional thresholding methods and the 4 band DT CWT with the improved thresholding proposed in this paper are superior to other several traditional thresholding de-noising methods in many aspects, such as smoothness, remaining the geometrical characteristics of the original test signal. Although the coefficient β could be chosen flexibly, it must be chosen by trial-and-error according to the needs of practice. So it is valuable to study how to find appropriate coefficient β rapidly and exactly.

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